QUADRATIC EQUATION

Form, Roots & Nature of Roots

Quadratic Equation : $f(x) = ax^2 + bx + c = 0$

Roots of Quadratic Equation	$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$b^2 - 4ac$ is called the discriminant or D which tells about nature of roots

- $D > 0$: Real and Distinct Roots
- $D = 0$: Real and Equal Roots
- $D < 0$: Imaginary Roots

If α & β are roots of the	$f(x) = ax^2 + bx + c = 0$
• Sum of Roots	◦ $\alpha + \beta = -b/a$
• Product of Roots	◦ $\alpha\beta = c/a$
• Difference of Roots	◦ $|a-b| = \sqrt{D}/|a|$

- Please note : $a\alpha + b = -c/\alpha$ & $a\beta + b = -c/\beta$

For two different Equations having discriminates

• $D1 + D2 \geq 0$	Atleast one equation has real roots
• $D1 + D2 < 0$	Atleast one equation has imaginary roots

- Quadratic Equation using roots : $a(x-\alpha)(x-\beta) = 0$

Condition for Irrational Roots

- D must not be a perfect square
- a,b,c are Rational Numbers
  - then, Roots are irrational are conjugate surds

If Coefficients are rational, then, irrational roots occur in pairs like **$p + \sqrt{q}$ & $p - \sqrt{q}$**

If Coefficients are real, then, imaginary roots occur in pairs like **$p + iq$ & $p - iq$**

Newton's Formula

For, $f(x) = ax^2 + bx + c = 0$ having roots α & β and given $S_n = \alpha^n \pm \beta^n$

Then, According to Newton's formula $x^2 \rightarrow S_n$ & $x^2 \rightarrow S_{n-1}$

Eqn changes to	$a.S_n + b.S_{n-1} + S_{n-2} = 0$

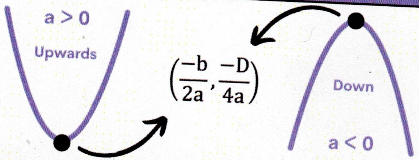
Common Roots

Equations : $ax_1^2 + bx_1 + c_1 = 0$ & $ax_2^2 + bx_2 + c_2$

Both Roots common	$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$
Exactly one Root Common	$\frac{b_1c_2 - b_2c_1}{a_2c_1 - a_1c_2} = \frac{a_2c_1 - a_1c_2}{a_1b_2 - a_2b_1}$

- If a = b = c = 0 in a quadratic equation, then it becomes an **Identity** with ∞ **roots**
- Other statement, If any polynomial has more than 'n' roots, it becomes an Identity **(Coefficients = 0)**

Graph of Quadratic Equation = Parabolic

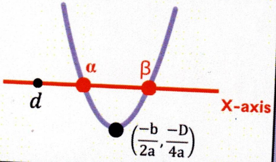


Conditions for roots to be greater than number 'd'

$a.f(d) > 0$

$\frac{-b}{2a} > d$

$D \geq 0$

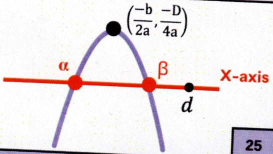


Conditions for roots to be less than number 'd'

$a.f(d) > 0$

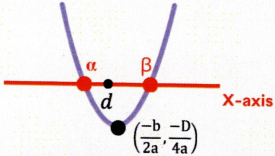
$\frac{-b}{2a} < d$

$D \geq 0$



Conditions for one root to be less than 'd' and other greater than 'd'

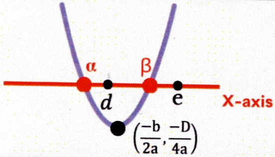
$a.f(d) < 0$



Exactly one root lies between (d,e) when d < e

$a \neq 0$

$f(d).f(e) < 0$



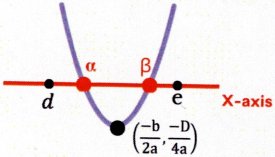
Both roots are confined between d & e (d < e)

$a.f(d) > 0$

$a.f(e) > e$

$D \geq 0$

$d < \frac{-b}{2a} < e$



One root greater than e, other less than d (d<e)

$a.f(d) < 0 \;\; \& \;\; a.f(e) < e$